Project: Apollo 13 Re-Entry

Unit: Differential Equations (AP Calc BC Unit 7)

Topics: Solving Separable Differential Equations, Euler's Method

In this project, students verify NASA's trajectory calculations for Apollo 13's re-entry by solving differential equations, simulating parachute descent with Euler's Method, and writing a flight readiness memo determining if the trajectory is safe.

Introduction:

You are part of NASA's trajectory verification team. A trajectory has been computed for Apollo 13's return, but before it is uploaded to the spacecraft, your job is to verify that the physics model and numerical approximations are accurate enough to guarantee a safe re-entry. Failure means the spacecraft will either burn up or bounce off the atmosphere, missing the Earth.

Part I: Verifying the Gravity Model

The spacecraft's velocity v(x) (in ft/s) as a function of altitude x (ft) is modeled by the equation:

$$\frac{dv}{dx} = \frac{-gR^2}{v(R+x)^2}$$

where $g = 32\ ft/s^2$ and $R = 20{,}900{,}000$ ft.

At Earth's entry interface, $x_0 = 400,000$ ft and $v_0 = 36,100$ ft/s

A. Derive the velocity model

Derive the velocity model for the spacecraft by integrating the differential equation to find the function v(x).

B. Predicted re-entry speed

A safe atmospheric corridor begins at $x_0 = 300,000$ft. Use the model you found in Part A to compute $v_0 = v(300,000)$. This is the velocity NASA plans to rely on when Apollo 13 hits the upper atmosphere. The head shield is rated for a maximum of 36,400 ft/s. Is the velocity you calculated safe?

C. Quantitative Error Propagation

Navigation data is not perfect. Suppose the computed entry velocity is off by $dv = \pm\ 2$ ft/s. This tiny uncertainty might be translated into a positional error.

1. Using your model v(x), compute $\frac{dx}{dv}$ at x = 300,000 (hint: you will need to invert $\frac{dv}{dx}$). Then, estimate the predicted entry altitude: $dx \approx \frac{dx}{dv} dv$.
2. Interpret your result: how far could Apollo 13's atmospheric entry point be shifted vertically in its predicted position because of a 2 ft/s velocity error?

Part II: The Parachute Descent

Once the spacecraft survives re-entry, it enters the lower atmosphere. At an altitude of 24,000 ft, the Drogue Parachutes deploy to stabilize the craft.

The Apollo Guidance Computer (AGC) could not solve the complex fluid dynamics of air resistance analytically. Instead, it predicts velocity using methods like Euler's method.

During this phase, the downward acceleration is determined by gravity (32) minus air resistance (drag). The velocity satisfies:

$$\frac{dv}{dt} = 32 - 0.15v, \text{ with } v(0) = 400 \text{ ft/s.}$$

A. AGC numerical simulation

Using Euler's Method with a step size $\Delta t = 0.5$ seconds, estimate the velocity of the spacecraft at t = 2 seconds. Use the update rule $v_{new} = v_{old} + \frac{dv}{dt}\Delta t$ and present your work in a table for t = 0, 0.5, 1.0, 1.5, and 2.0. Round all values to three decimal places.

B. Accuracy check

The exact solution to this differential equation yields a velocity of v(2) = 351.625 ft/s. Calculate the absolute error of your Euler approximation. Why might this type of error have been acceptable to NASA? Why is the error relatively small in this case?

C. Computational Constraints:

In the Apollo era, the AGC had very limited memory. Explain why NASA engineers would choose Euler's Method (from Part II) instead of storing a table of pre-calculated solutions or having the AGC calculate solutions live.

Flight Readiness Certification

Now that you have verified the calculations, write a short 1-2 paragraph memo to NASA Flight Control certifying whether the Apollo 13 trajectory is flight-ready.

Make sure to include:

1. The velocity you calculated at the entry altitude x = 300,000 ft, and whether or not it is within the safe window (36,100 and 36,400 ft/s)
2. How reliable this prediction is, by referencing your calculations in part I-C and II-B
3. Your decision on whether this trajectory is flight-ready or not